

18. EXERCISES

Section 1.

- (E1.1) Prove that if G is a finite simple abelian group, then $G \cong C_p$, the cyclic subgroup of order p , where p is a prime.

Section 2.

- (E2.1) Prove that **Set**, **Pfn**, **Grp**, **Top** and \mathbf{Vect}_K are categories.
- (E2.2) Which categories in Example 1 are (full) subcategories of some other category in Example 1?
- (E2.3) Complete the definition of **Digraph** and prove that it is a category.
- (E2.4) Give the ‘right’ definition of the category **Graph** corresponding to graphs that are not necessarily simple, i.e. which may have multiple edges between vertices.
- (E2.5) Prove that \mathbf{Vect}_K and $\mathbf{IVect}_{\mathbb{R}}$ are categories.
- (E2.6) Prove that **G-Set** is a category.
- (E2.7) Prove that Example 5 yields a category.
- (E2.8) What is $\text{Aut}(X)$ when X is an object in **Top**?
- (E2.9) Show that
- (1) an isomorphism is monic and epic;
 - (2) if \mathbf{C} is a category of structured sets (so that each arrow is carried by a total function between the carriers of the two objects), then

injective \implies monic, and surjective \implies epic.
 - (3) Show that epic does not imply surjective in **Ring**.
 - (4) Show that bijective does not imply isomorphism in **Top**.
- (E2.10) What are automorphisms in **Graph**? Can you see why one needs a different definition in this context?