EXERCISE SHEET 5

(E99) $\mathrm{SU}_2(q) \cong \mathrm{SL}_2(q)$ and, moreover, the action of $\mathrm{SU}_2(q)$ on the set of points of the associated polar space is isomorphic to the action of $\mathrm{SL}_2(q)$ on the set of points of $\mathrm{PG}_1(q)$.

(E100) Prove that if $\beta(x,y) = 0$, then there exists z with $\beta(x,z), \beta(y,z) \neq 0$.

(E101) Prove that if $\beta(x,y) \neq 0$, then there exists z with $\beta(x,z) = \beta(y,z) = 0$. .

(E104) Complete the proof of Lemma 61 in lectures.

(E105) Prove that $\mathrm{SU}_4(2)$ is generated by transvections.

(E107) Prove that $\mathrm{PSU}_3(2) \cong E \rtimes Q$ where E is an elementary abelian group of order 9 and Q is a quaternion group of order 8.

(E109) Prove that

$$|\mathrm{SO}_n^\varepsilon(q)| = |\mathrm{PO}_n^\varepsilon(q)| = \frac{1}{(2, q-1)} |\mathrm{O}_n^\varepsilon(q)|.$$

(E111) $\mathrm{SO}_n^\varepsilon(q)$ contains a transvection if and only if q is even.

(E112) Prove that the definition of $\Omega_n^+(q)$ is well-defined when $\varepsilon = +$, by showing that in the natural action of G on $\mathcal{U}_r$, the set of maximal totally singular subspaces, any reflection acts as an odd permutation on $\mathcal{U}_r$.