Conway Groupoids

Nick Gill

7th May 2015

Joint with
Gillespie and Semeraro (Bristol); Nixon (Lancaster); Praeger (UWA).

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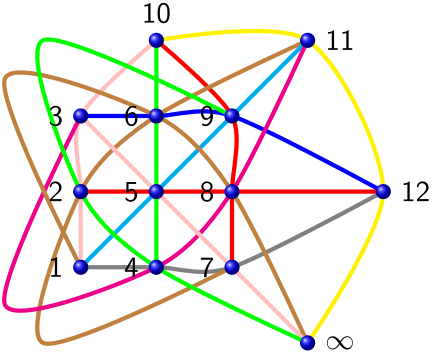

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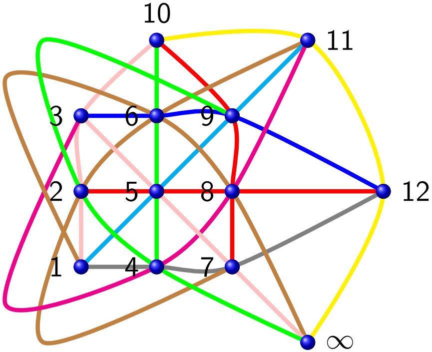

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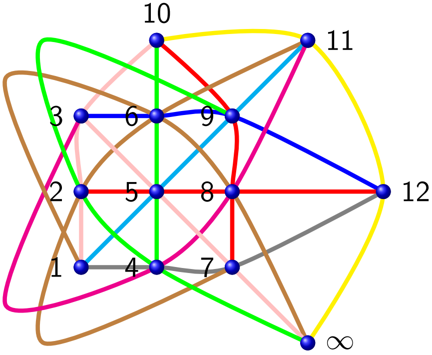

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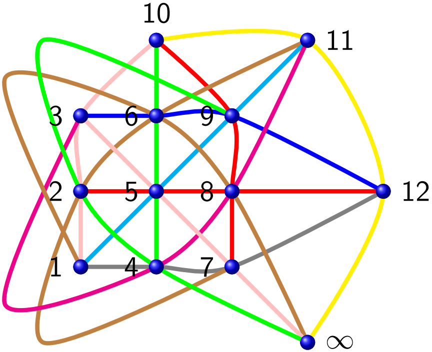

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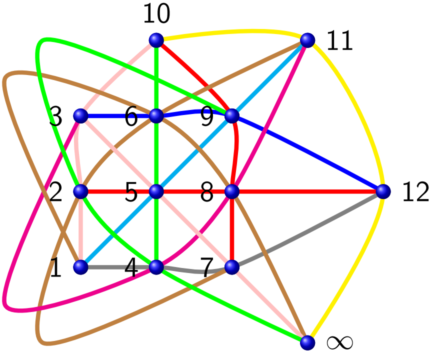
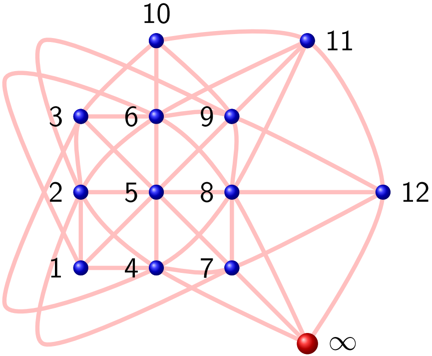


- 13 lines, 13 points;
- every line contains 4 points;
- every pair of points is connected by exactly one line;
- every pair of lines intersects in exactly one point.

The M_{13} game



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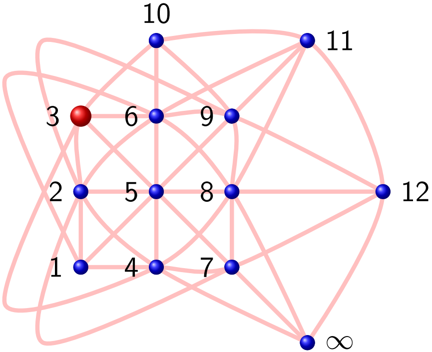

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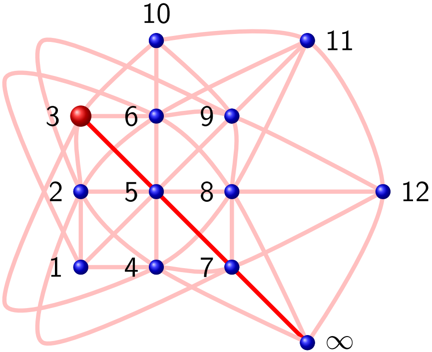

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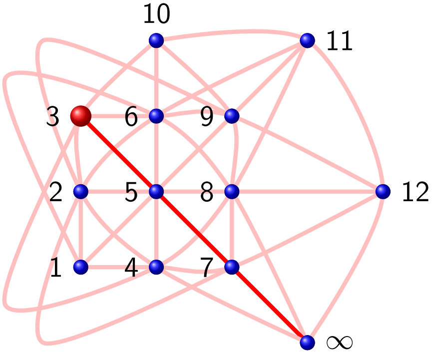

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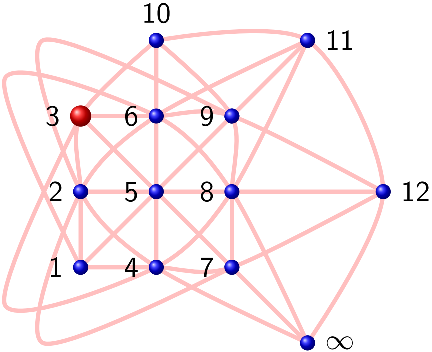
A move sequence



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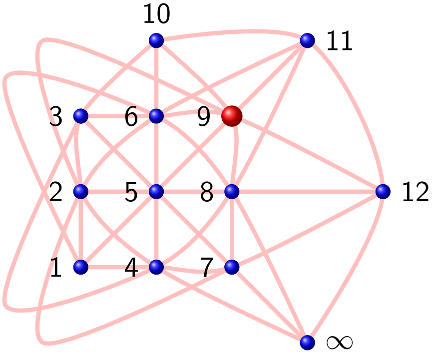

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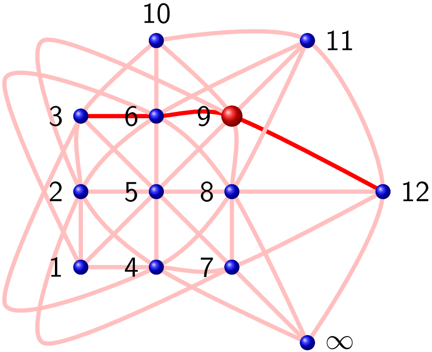

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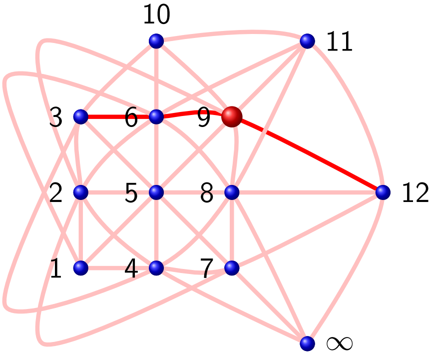

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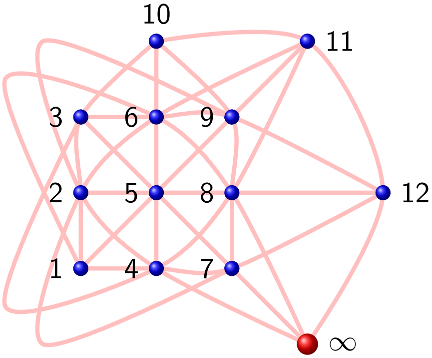

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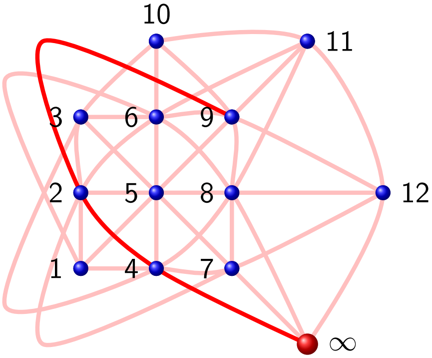

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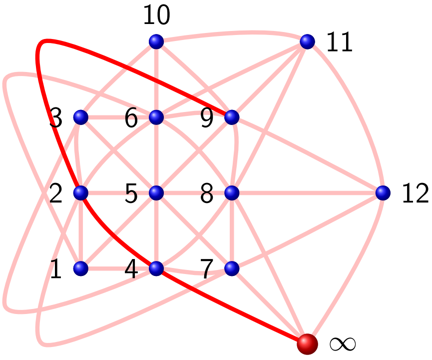

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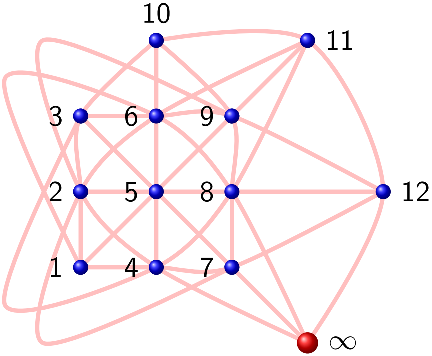

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- $\mathcal{L}_\infty(\mathbb{P}_3)$ is a union of cosets of $\pi_\infty(\mathbb{P}_3)$; it is also called M_{13}.

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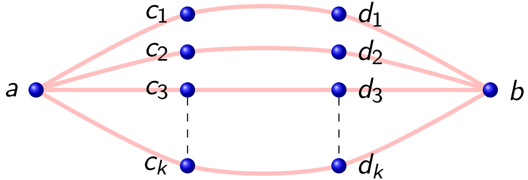

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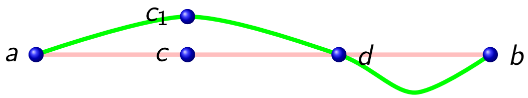

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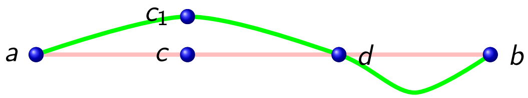

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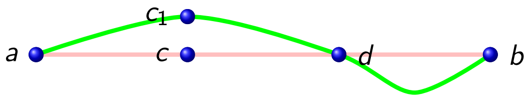

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We call π_∞ the **puzzle group** of the geometry, and $\mathcal{L}_\infty$ the **Conway groupoid**.

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- A supersimple $2-(n,4,\lambda)$ design $\mathcal{D}$ satisfies Axiom 1 and Axiom 2.
- We wish to calculate $\pi_\infty(\mathcal{D})$ and $\mathcal{L}_\infty(\mathcal{D})$ for all possible supersimple designs $\mathcal{D}$.

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- Thus $2(n - 1 - 2\lambda) \leq n - 1$.

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Corollary

If $n > 144\lambda^2 + 120\lambda + 26$, then $\pi_\infty(\mathcal{D})$ contains $\mathrm{Alt}(n-1)$.

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- **Fact**: If $\lambda = 2$, then
  1. $n = 10$ and $\pi_\infty(\mathcal{D}) = \mathrm{O}_4^+(2)$, or
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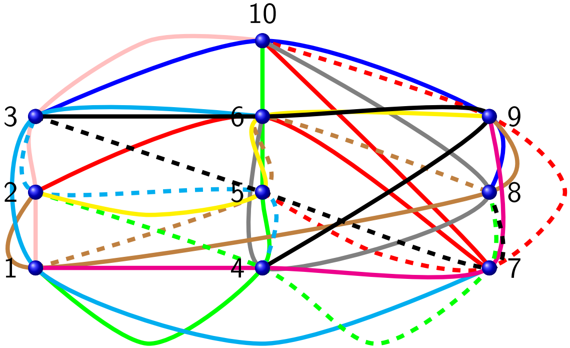

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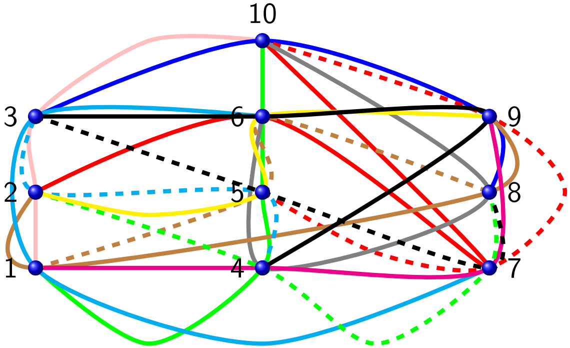

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Properties A and B imply that $\mathcal{L}_\infty(\mathcal{D})$ is a subgroup of $\mathrm{Sym}(\Omega) = \mathrm{Sym}(n)$.

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  a class of 3-transpositions in $\mathcal{L}_\infty(\mathcal{D})$ that generate $\mathcal{L}_\infty(\mathcal{D})$.
- One of the following holds:
  1. $n = 2\lambda + 2 = 2^a$, $\mathcal{D}$ is a Boolean design, $\pi_\infty(\mathcal{D})$ is trivial, and $\mathcal{L}_\infty(\mathcal{D})$ is regular and elementary abelian;

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Assume that $\mathcal{D}$ is a supersimple $2-(n,4,\lambda)$ design satisfying Properties A and B.

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$$\{[a,b] \mid a,b \in \Omega\}$$

  a class of 3-transpositions in $\mathcal{L}_\infty(\mathcal{D})$ that generate $\mathcal{L}_\infty(\mathcal{D})$.
- One of the following holds:
  1. $n = 2\lambda + 2 = 2^a$, $\mathcal{D}$ is a Boolean design, $\pi_\infty(\mathcal{D})$ is trivial, and $\mathcal{L}_\infty(\mathcal{D})$ is regular and elementary abelian;
  2. $n > 2\lambda + 2$, $\mathcal{L}_\infty(\mathcal{D})$ acts 2-primitively on Ω and $\pi_\infty(\mathcal{D})$ is the stabilizer of the point ∞ in this action.

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In particular $\mathrm{Sp}_{2m}(2)$ and $2^{2m}.\mathrm{Sp}_{2m}(2)$ occur as Conway groupoids.

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- What is the role of to-graphs? Can groupoids be constructed naturally from two-graphs, without going via designs?
- What are the connections to Bruhat-Tits buildings?
- If $\pi_\infty(\mathcal{D})$ is primitive, is it true that $\mathcal{L}_\infty(\mathcal{D})$ is either M_{13} or a group?

Thank you for listening!